Journal of Earth Science, *Vol. 30*, *No. 2*, *p. 422–428*, *April 2019* *ISSN 1674-487X*
Printed in China
https://doi.org/10.1007/s12583-018-0834-0

Formulation of Determining the Gravity Potential Difference Using Ultra-High Precise Clocks via Optical Fiber Frequency Transfer Technique

Ziyu Shen[1], **Wen-Bin Shen***[2, 3], **Zhao Peng**[1], **Tao Liu**[4], **Shougang Zhang**[4], **Dingbo Chao**[2]

1. *School of Resource and Environment*, *Hubei University of Science and Technology*, *Xianning* 437100, *China*
2. *Time and Frequency Geodesy Research Center*, *School of Geodesy and Geomatics*, *Department of Geophysics*, *Key Laboratory of Geospace Environment and Geodesy of the Ministry of Education*, *Wuhan University*, *Wuhan* 430079, *China*
3. *State Key Laboratory of Information Engineering in Surveying*, *Mapping and Remote Sensing*, *Wuhan University*, *Wuhan* 430079, *China*
4. *National Time Service Center* (*NTSC*), *Chinese Academy of Sciences*, *Xi'an* 710600, *China*

Ziyu Shen: https://orcid.org/0000-0002-7261-1068; Wen-Bin Shen: https://orcid.org/0000-0002-9267-5982

ABSTRACT: Based on gravity frequency shift effect predicted by general relativity theory, this study discusses an approach for determining the gravity potential (geopotential) difference between arbitrary two points *P* and *Q* by remote comparison of two precise optical clocks via optical fiber frequency transfer. After synchronization, by measuring the signal's frequency shift based upon the comparison of bidirectional frequency signals from *P* and *Q* oscillators connected with two optical atomic clocks via remote optical fiber frequency transfer technique, the geopotential difference between the two points could be determined, and its accuracy depends on the stabilities of the optical clocks and the frequency transfer comparison technique. Due to the fact that the present stability of optical clocks achieves 1.6×10^{-18} and the present frequency transfer comparison via optical fiber provides stabilities as high as 10^{-19} level, this approach is prospective to determine geopotential difference with an equivalent accuracy of 1.5 cm. In addition, since points *P* and *Q* are quite arbitrary, this approach may provide an alternative way to determine the geopotential over a continent, and prospective potential to unify a regional height datum system.

KEY WORDS: gravity frequency shift, optical fiber frequency transfer, optical clock, gravity potential.

0 INTRODUCTION

Geopotential (gravitational potential plus centrifugal force potential) is a basic entity in physics and geoscience, plays a key role in various research fields and has broad applications (Li et al., 2016; Tenzer and Bagherbandi, 2016; Hofmann-Wellenhof and Moritz, 2006), and is the foundation of the definitions of the geoid and world height system. One direct application of geopotential provides orthometric height, the height above the geoid that is a closed equi-geopotential surface nearest to the mean sea level.

The conventional way to determine the geopotential is based on the "leveling plus gravimetry" approach (Hofmann-Wellenhof and Moritz, 2006), which has at least two disadvantages: (a) the error is accumulated with the increase of the leveling propagation measurements, and (b) it is difficult or impossible to transfer the orthometric height with high accuracy between two points located in mountainous areas or continents separated by sea. To overcome the drawbacks existing in the conventional approach, Bjerhammar (1985) put forward an idea to determine the geopotential and orthometric height using precise clocks via portable clock comparison, which is based on the Einstein's general relativity theory (GRT): precise clocks run quicker at a position with higher potential. This approach is referred to as the clock transportation approach (Mai, 2013; Shen et al., 2009). Equivalently, an approach based on the gravitational redshift effects of GRT was proposed (Shen et al., 1993), which is referred to as the gravity frequency shift approach (GFSA) (Shen et al., 2011, 2009, 1993; Shen, 1998). The main idea of GFSA is stated as follows.

According to GRT, when a receiver at point Q receives a light signal emitted from point P, the receiving frequency is different from the innate frequency at Q due to geopotential difference between these two points (Shen et al., 2009; Shen, 1998; Soffel et al., 1988a, b; Weinberg, 1972). Exactly to say, if there are two precise clocks located at points P and Q with different geopotentials, the gravity frequency shift of the signal transmitting between these two points can be expressed as follows (Lion et al., 2017; Shen Z Y et al., 2017, 2016; Flury, 2016; Mai, 2013; Shen W B et al., 2011, 1993; Chou et al., 2010a, b; Shen W-B, 1998; Weinberg, 1972; Pound and Snider, 1965).

*Corresponding author: wbshen@sgg.whu.edu.cn

Manuscript received February 5, 2017.
Manuscript accepted August 20, 2017.

Shen, Z. Y., Shen, W.-B., Peng, Z., et al., 2019. Formulation of Determining the Gravity Potential Difference Using Ultra-High Precise Clocks via Optical Fiber Frequency Transfer Technique. *Journal of Earth Science*, 30(2): 422–428. https://doi.org/10.1007/s12583-018-0834-0. http://en.earth-science.net

$$\frac{f_Q - f_P}{f} = \frac{\Delta f_{PQ}}{f} = \frac{\Delta W_{PQ}}{c^2} = -\frac{W_Q - W_P}{c^2} \tag{1}$$

where f_P is the emitting frequency of a signal from point P, f_Q is the innate frequency of the clock at point Q, c is the speed of light in vacuum, W_P and W_Q are the geopotentials at P and Q respectively. We note that the geopotentail, W, is the sum of the Earth's gravitational potential and the centrifugal force potential caused by the Earth rotation. In Eq. (1), there appears a negative sign, which is due to the fact that in physical geodesy the geopotential has opposite signature as it has in physics.

Equation (1) means that the oscillation frequency of the clock located at a lower position with smaller orthometric height (which is the height above the geoid) is smaller with respect to the clock located at a higher position. Then, by directly comparing the innate frequency with the receiving frequency, there will be a gravity frequency shift Δf_{PQ} between these two clocks (Müller et al., 2010; Shen, 1998). Inversely, based on Eq. (1), if the frequency shift Δf_{PQ} is measured with an accuracy of 1×10^{-18} level, the geopotential difference between P and Q could be determined with an accuracy of equivalent one-centimeter level.

Various experiments confirmed that the gravitational redshift effect or gravity frequency shift Eq. (1) was correct to certain accuracy level (Chou et al., 2010a, b; Müller et al., 2010; Turneaure et al., 1983; Katila and Riski, 1981; Vessot et al., 1980; Vessot and Levine, 1979; Snider, 1972; Pound and Snider, 1965; Pound and Rebka, 1960a, b, 1959). For instance, after it was confirmed by Mössbauer experiment with a relative accuracy of 1×10^{-2} (Pound and Snider, 1965; Pound and Rebka, 1960a, b, 1959), an accuracy of 7×10^{-5} was obtained based on a system consisting of ground stations and on-board a hydrogen clock in a rocket (Vessot et al., 1980; Vessot and Levine, 1979). Recently, Müller et al. (2010) declared that their experimental results show that Eq. (1) is correct at 7×10^{-9} level, which implies that, under the assumption that present clocks with sufficient accuracy are used and the environmental noises are neglected, only an error of 7×10^{-3} mm in equivalent height in 1 km could be introduced if GRT does not strictly hold (this error could achieve 7 mm in a height difference of 1 000 km between two points). However, their conclusions are under debate. It is by far not clear, and in fact very doubtable, that a Compton frequency of an atom establishes a more precise clock. Nevertheless, for the purpose of measuring the geopotential via GFSA in free space, we may assume that the gravity frequency shift Eq. (1) holds correct.

The real realization of the GFSA depends on the error control during the signal's propagation in free space and the stability (uncertainty) of the clocks used for comparing the time-frequency signals. If the frequency shift measurement accuracy achieves 1×10^{-18}, the accuracy of the determined geopotential and orthometric height can achieve 1 cm (Shen et al., 1993). In fact, about 10 years ago, scientists predicted that optical clocks could achieve a stability and accuracy of 10^{-18} to 10^{-19} level (Akatsuka et al., 2008; Ludlow et al., 2008; Rosenband et al., 2008; Diddams et al., 2004, 2001; Ma et al., 2004; Ye et al., 2003), which has been realized to date. In 2011 and 2012 optical clocks with a stability of around 10^{-17} were successively generated (Huntemann et al., 2012; Madej et al., 2012; Katori, 2011), and later optical clocks with stability of 1.6×10^{-18} in seven hours' average or with similar accuracy level were created (Ushijima et al., 2015; Bloom et al., 2014; Hinkley et al., 2013). Hence, concerning the present achievements of time and frequency science, GFSA may provide direct geopotential difference and orthometric height difference measurements at the accuracy level of 1.5 cm if the environmental noise influences are neglected.

However, at present it is likely quite difficult to realize precise measurement of geopotential directly using GFSA, because environmental influences (e.g., atmosphere and ionosphere influences) are difficult to control, which may largely contaminate the electromagnetic signals (simply light signals or signals hereafter for convenience) propagating in free space. To overcome this drawback, Shen and Peng (2012) proposed an idea: one may determine the geopotential difference based upon the optical fiber frequency transfer technique and optical clocks, which is for convenience referred to as geopotential-difference optical-fiber frequency transfer (GOFT) (Shen, 2013a, b). To date, the quick development of the remote frequency comparison techniques via optical fiber (Predehl et al., 2012; Marra et al., 2011; Grosche et al., 2009; Kéfélian et al., 2009; Jiang et al., 2008; Newbury et al., 2007a, b) provides prospective potential to directly measure the geopotential differences between two points connected by optical fiber using optical clocks at two remote ends. The advantage in transmitting light signals via optical fiber but not in free space lies in that the former can cancel out significant environment noises, which otherwise will greatly contaminate the signals. Various authors confirmed that remote optical fiber frequency transfer technique could provide laser-based frequency comparison between two stations separated by distances from 50 to 600 km at the uncertainty levels from 10^{-18} to 10^{-19}. For instance, after an uncertainty 6×10^{-19} in 100 s via optical frequency transfer over 251 km of optical fiber length was realized (Newbury et al., 2007b), an uncertainty 1×10^{-19} in 8 hrs via optical frequency transfer with fiber length of 142 km was achieved (Grosche et al., 2009), further the laser-based frequency transfer via a 108 km-long optical fiber with an uncertainty below 1×10^{-19} in 2.8 hrs was achieved (Kéfélian et al., 2009). A recent study (Predehl et al., 2012) demonstrated that the uncertainty of the optical fiber frequency transfer comparison between two laboratories separated by a distance of 600 km reaches 1×10^{-18} in less than 17 min, and for a longer integration time the frequency comparison stability achieves 4×10^{-19}, which could serve as regional (say a Europe-wide) optical frequency dissemination network. Hence, comparing with the present stability level 1×10^{-18} of optical clocks (Ushijima et al., 2015; Bloom et al., 2014; Hinkley et al., 2013), the frequency comparison stability is high enough for the purpose of determining the geopotential and orthometric height with the accuracy of one-centimeter level.

Here we note that the concept and methodology of the optical fiber frequency transfer was proposed around 30 years ago (e.g., Primas et al., 1988), and the remote optical fiber frequency comparison with stability (and accuracy) of 10^{-18} level or above was realized in recent 10 years by various groups internationally (e.g., Wada et al., 2015; Raupach et al., 2014; Droste et al., 2013; Lopez et al., 2013, 2012; Raupach and Grosche, 2013; Predehl et al., 2012; Marra et al., 2011; Grosche et al., 2009; Kéfélian et al., 2009; Jiang et al., 2008; Newbury et al., 2007a, b). These studies mainly focused on the purpose of, for instance, GRT test, precise measurements of physical constants (e.g., fine structure constant), even detection of gravitational wave. Shen and Peng (2012) firstly proposed the approach to determine the geopotential difference between two remote points (stations) using optical fiber frequency transfer technique. Takano et al. (2016) also discussed the geopotential measurements with synchronously linked optical lattice, and recently, relevant experimental results have been reported (Grotti et al., 2018; Lion et al., 2017; Lisdat et al., 2016).

Previously, Shen and Peng (2012) assumed that when light signals transmit in optical fibers, their frequencies have the same nature as the light signals transmitting in free space. In fact, this assumption is not needed (Shen, 2013a, b). Chou et al. (2010a, b) executed an excellent experiment to demonstrate that the gravity frequency shift Eq. (1) holds also for light signals transmitting in optical fibers. Recently, addressed to a clock network in geodesy, based on remote optical fiber frequency transfer Lisdat et al. (2016) and Grotti et al. (2018) further confirmed Eq. (1). For the purpose of actual applications and for further improvements of the previous investigations (Shen, 2013a, b; Shen and Peng, 2012), the present study focuses on the formulation of how to practically realize the GOFT.

After an introductory context (Section 0), Section 1 provides a formulation of gravity frequency shift determination using remote optical fiber frequency comparison technique. Section 2 discuses how to determine the geopotential and orthometric height based on the measured gravity frequency shift. Section 3 briefly summarizes the main context of this study, suggests its potential application in regional height system unification and provides relevant discussions.

1 GRAVITY FREQUENCY SHIFT MEASUREMENT VIA OPTICAL FIBER FREQUENCY TRANSFER COMPARISON TECHNIQUE

To precisely determine the gravity frequency shift between two stations P and Q, we execute the following procedures: (1) Two optical clocks C_P and C_Q are synchronized by frequency (namely adjusted to the same frequency) at beginning at same site (station), for instance at station P; (2) clocks C_P and C_Q are fixed at stations P and Q, respectively, and they are connected by two identical optical fibers; (3) using the optical fiber frequency transfer comparison technique as described below, one can determine the frequency shift $\Delta f_{PQ}=f_Q-f_P$ between P and Q. Suppose Q is arbitrary, if setting point P on the geoid or at a datum point (with known geopotential), the geopotential at point Q can be determined.

Suppose two optical clocks are located at points P and Q which are connected by optical fibers F_1 and F_2 (see Fig. 1). To precisely compare the innate frequency with the receiving frequency, different kinds of error sources should be carefully considered. For instance, the changes of the optical path length due to acoustic noise and temperature fluctuations limit the stability and accuracy of the transmitted frequency, and such kinds of noises should be effectively controlled (Predehl et al., 2012; Newbury et al., 2007b). The most serious error is Doppler effect, which is caused by the fiber length variation induced by the mechanical perturbations and temperature variations (Predehl et al., 2012; Ma et al., 1994). To cancel the Doppler effect, Doppler cancellation technique (Ye et al., 2003; Ma et al., 1994) can be applied, namely, bidirectional identical fibers which transmit bidirectional light signals can be adopted (Predehl et al., 2012; Newbury et al., 2007a). Another problem is that the signals will attenuate during their transmission in fiber. One possible solution is connecting optical fibers with erbium-doped fiber amplifiers (EDFAs) to overcome inherent attenuation of the transmitting signals (Predehl et al., 2012; Newbury et al., 2007a, b). The longer the fiber, the more EDFAs are needed. In addition, two fiber Brillourin amplifiers (FBAs) are required at both ends to guarantee coherent and fully transparent transmission (Guena et al., 2012; Predehl et al., 2012; Newbury et al., 2007a, b; Ma et al., 1994).

Now, station P emits signal toward station Q via fiber 1 (F_1), and station Q observes a frequency shift, denoted as "observation-at-Q", $\Delta f_{\text{Obs-at-}Q}$, which can be expressed as

$$\Delta f_{\text{Obs-at-}Q} = \Delta f_{PQ} + \Delta f_{\text{DPL1}} + \Delta f_{\text{F}_1} + \Delta f_{\text{Ram1}} \tag{2}$$

where Δf_{PQ} is the gravity frequency shift (caused by the geopotential difference), Δf_{DPL1} is Doppler effect during the signal's propagation in F_1, Δf_{F_1} is the sum of various errors caused by circumstances, and Δf_{Ram1} is a random error. Simultaneously station Q emits signal towards station P via fiber 2 (F_2), and similarly we have

$$\Delta f_{\text{Obs-at-}P} = \Delta f_{QP} + \Delta f_{\text{DPL2}} + \Delta f_{\text{F}_2} + \Delta f_{\text{Ram2}} \tag{3}$$

Since F_2 is identical with F_1, and since stations P and Q emit signals simultaneously, we may expect that $\Delta f_{\text{DPL2}}=\Delta f_{\text{DPL1}}\equiv \Delta f_{\text{DPL}}$ and $\Delta f_{\text{F}_2}=\Delta f_{\text{F}_1}\equiv\Delta f_{\text{F}}$. Noting that $\Delta f_{PQ}= -\Delta f_{QP}$, subtraction of Eqs. (2) and (3) provides $\Delta f_{\text{Obs-at-}Q}-\Delta f_{\text{Obs-at-}P}=2\Delta f_{PQ}+\Delta f_{\text{Ram1}}-\Delta f_{\text{Ram2}}$, or

$$\Delta f_{PQ} = \frac{\Delta f_{\text{Obs-at-}Q} - \Delta f_{\text{Obs-at-}P}}{2} - \frac{\Delta f_{\text{Ram1}} - \Delta f_{\text{Ram2}}}{2} \tag{4}$$

Exchanging data between P and Q, we can determine the gravity frequency shift based on Eq. (4). By multi-times observations, taking simple average, we may improve the result, because the random terms could be greatly reduced or cancelled.

To realize the time synchronization, or properly say, quasi-synchronization, we take the following scheme. At the time that Q receives the frequency signal S_P (see Fig. 1), it immediately emits frequency signal S_Q. After P receives the signal S_Q, the time duration, Δt, of the signal's propagation in F_1 (or F_2) can be estimated. Now, P emits two successive signals S_{P0} (initial signal) and S_P with interval Δt; at the moment receiving the initial signal S_{P0}, Q immediately emits frequency signal S_Q towards P. In this way, the frequency signals S_P and S_Q are emitted simultaneously. Here, the delay between the emission of S_P and S_Q can be neglected.

Since the random noises introduced by frequency transfer comparison via optical fibers (with length of at least 920 km) could be controlled to an uncertainty below 4×10^{-19} (Predehl et

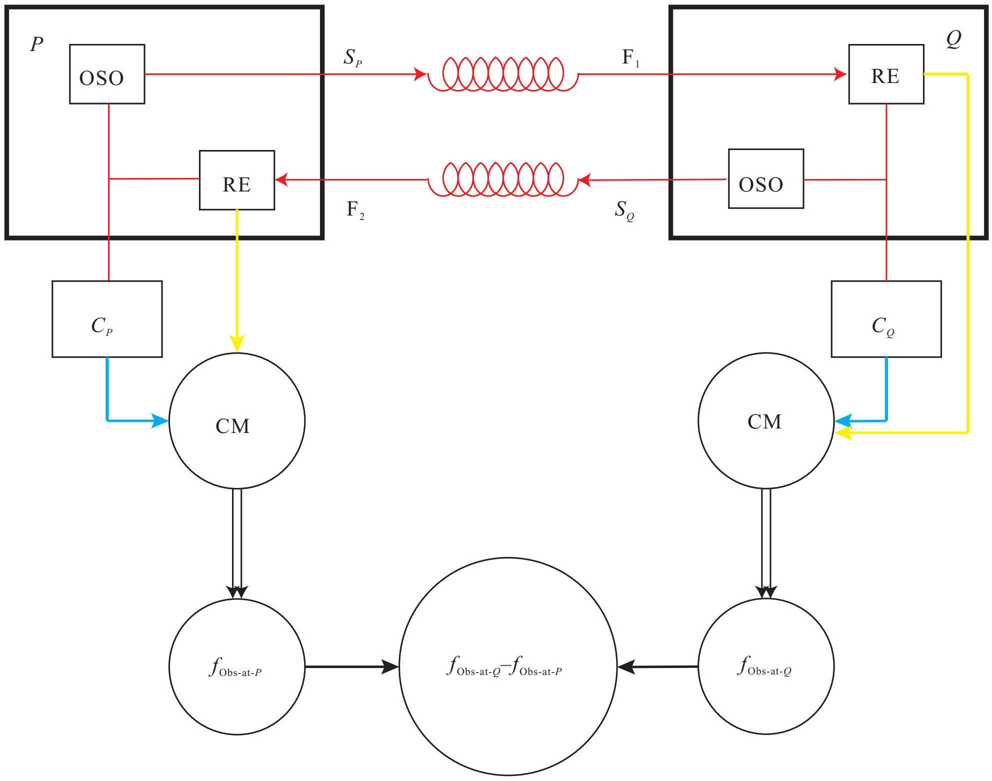

Figure 1. Points P and Q denote two stations separated by a distance. S_P and S_Q are frequency signals (propagating in fibers F_1 and F_2, respectively) emitted by optical signal oscillators (OSO) which are connected with optical clocks C_P and C_Q at stations P and Q, respectively. The receiver RE at P (or Q) receives a frequency signal from Q (or P), and a frequency shift observation $\Delta f_{\text{Obs-at-}P}$ (or $\Delta f_{\text{Obs-at-}Q}$) is obtained by comparison measurement (CM). F_1 and F_2 are two identical optical fibers, with a number of optical amplifiers to preserve the signal power and coherence (modified after Predehl et al., 2012).

al., 2012), if the accuracies of the optical clocks C_P and C_Q could achieve 1×10^{-18} level, one could determine the frequency shift Δf_{PQ} with a stability level of 1×10^{-18}, which is equivalent to the height variation of 1 cm. Then, based on Eqs. (1) and (4), the geopotential difference W_Q–W_P between P and Q can be determined.

2 DETERMINATION OF GEOPOTENTIAL AND ORTHOMETRIC HEIGHT DIFFERENCES

After the gravity frequency shift $\Delta f_{PQ}=f_Q-f_P$ between P and Q is measured based on Eqs. (2) to (4), one can determine the corresponding geopotential difference $\Delta W_{PQ}=W_Q-W_P$ based on Eq. (1). In the sequel, we describe how to determine the orthometric height based on the determined geopotential difference ΔW_{PQ}.

Without loss of generality, we may suppose that point P is on the geoid (or at a datum with known orthmetric height), then the orthometric height at point Q is determined based on the following integral formula (Hofmann-Wellenhof and Moritz, 2006)

$$W_Q-W_0=-\int_{g_{O(Q)}}^{g_Q} g dh \tag{5}$$

where g_Q is the gravity at point Q and $g_{O(Q)}$ the gravity at the point $O(Q)$ on the geoid corresponding to the point Q, where $O(Q)$ denotes the projection point on the geoid of the point Q along the plumb line. Here we note that, the gravity g is the sum of gravitation and centrifugal force (the latter is related to Earth rotation). The orthometric height is a geometric length measurement along the plumb line from the geoid at point O to the ground point Q. The plumb line is a curved line connecting the two points, at any point of which the straight line of the tangent vector coincides with that of the gravity vector. We also note that in Eq. (5), the upper and lower limits of integration, g_Q at point Q and $g_{O(Q)}$ at point $O(Q)$, correspond to the orthometric heights of H_Q and H_O, respectively.

Equation (5) can be solved only if the gravity along the plumb line from Q to $O(Q)$ is given. However, inside the Earth, we don't know exactly the gravity distribution, though we may use PREM model (Dziewonski and Anderson, 1981) to approximately determine its interior distribution. Mathematically, applying the mean value theorem, from Eq. (5) one obtains $W_Q-W_0=-\bar{g}H_Q$, or equivalently

$$H_Q=-\frac{W_Q-W_0}{\bar{g}} \tag{6}$$

where $\bar{g}$ is a "mean value" between g_Q at point Q and $g_{O(Q)}$ at point $O(Q)$, namely, $\bar{g}$ is the gravity at a point somewhere on the plumb line connecting the points Q and $O(Q)$. In practice, $\bar{g}$ could be approximately replaced by g_Q+0.042 4H (Heiskanen and Moritz, 1967), and then, Eq. (6) reads

$$H_Q=-\frac{W_Q-W_0}{g_Q+0.042\,4H} \tag{7}$$

where g_Q in gals (cm/s^2), H in km, and W in g.p.u (cm^2/s^2).

Combining Eqs. (1), (4) and (7), taking into account W_Q–

$W_0=W_Q-W_P=\Delta W_{PQ}$ (note that point P is on the geoid), the orthometric height of point Q is determined by the following formula

$$H_Q = -\frac{1}{g_Q + 0.042\,4H}\frac{\Delta f_{PQ}}{f} \quad (8)$$

To estimate the error caused by the frequency uncertainty δf_{PQ}, replacing g_Q by γ_Q or $\bar{\gamma}$, where γ_Q is the normal gravity at point Q and $\bar{\gamma}$ the average normal gravity over the ellipsoid WGS84, one obtains

$$\delta H_Q = \frac{1}{\bar{\gamma}}\frac{\delta f_{PQ}}{f} \approx 9.1\times 10^{15}\frac{\delta f_{PQ}}{f} \quad (9)$$

which implies that the accuracy in determining the orthometric height H_Q depends on the uncertainty of the measured frequency shift Δf_{PQ}, which further depends on the stabilities of the optical clocks and transmitting frequency comparison. Hence, due to present quick development of optical atomic clocks, it is prospective to realize one centimeter-level determination of the geopotential and orthometric height differences between arbitrary two points which are connected by optical fibers.

3 CONCLUSIONS AND DISCUSSIONS

To determine the geopotential difference between two points using precise optical clocks via remote optical fiber frequency transfer comparison technique, the key problem is to precisely determine the gravity frequency shift of light signals transmitting in optical fibers. Various experimental results showed that the measurement accuracy of the gravity frequency shift of transmitting signals via optical fibers could be controlled to the level of 10^{-18} if the stabilities of optical clocks achieve 10^{-18} level. Consequently it is prospective and potential to determine the geopotential difference and the corresponding orthometric height difference at the centimeter level between arbitrary two points connected by optical fibers using GOFT, if optical clocks with stabilities of 10^{-18} level are available.

Suppose two points P and Q are located at two arbitrary points in a connected continent (China and Europe). The geopotential difference of these two points might not be or difficult to be measured by conventional leveling plus gravimetry approach or could not be precisely determined by gravity model approach. However, if these points are connected by optical fibers (e.g. via intermediate stations), the geopotential difference can be precisely determined using GOFT under the condition that precise clocks are available. Based on this study, using as many as precise optical clocks, we may establish especially regional datum network of geopotential and orthometric height.

Ultra-highly precise optical clocks and recent quick development in transmitting frequency comparison technique may provide more accurate test of GRT and greatly contribute to geoscience community. For instance, based upon GOFT one may provide new result of testing GRT at the centimeter level in the absolute sense. The main idea is stated as follows. Choose two points A and B that are not far away from each other, with an orthometric height difference ΔH_{AB}. By conventional leveling and gravimetry the orthometric height difference ΔH_{AB} could be precisely determined, say with an accuracy level better than 1 cm. Now suppose the stability of the clocks used is at 1×10^{-18} level, which is sensitive to a height variation of 1 cm. If the measured height difference based upon GOFT is denoted as $\Delta H_{AB}^{\mathrm{Obs}}$, then the quantity $\Delta H_{AB}^{\mathrm{Obs}} - \Delta H_{AB}$ suggests the difference between the GRT prediction and the real observation.

As a further improvement of Shen and Peng (2012) and Shen (2013a, b), this study further suggests that determining the geopotential difference and the corresponding orthometric height difference between arbitrary two points using optical clocks via optical fiber frequency transfer technique is prospectively potential, and at the same time the realization of the GOFT may contribute to the unification of a regional height system (e.g., China height system and Europe height system) with high accuracy.

ACKNOWLEDGMENTS

We sincerely thank three anonymous reviewers, who's valuable comments and suggestions greatly improved the manuscript. This study was supported by the National Natural Science Foundation of China (Nos. 41631072, 41721003, 41574007, and 41429401), the Discipline Innovative Engineering Plan of Modern Geodesy and Geodynamics (No. B17033), the DAAD Thematic Network Project (No. 57173947), and the International Space Science Institute (ISSI) 2017–2019. The final publication is available at Springer via https://doi.org/10.1007/s12583-018-0834-0.

REFERENCES CITED

Akatsuka, T., Takamoto, M., Katori, H., 2008. Optical Lattice Clocks with Non-Interacting Bosons and Fermions. *Nature Physics*, 4(12): 954–959. https://doi.org/10.1038/nphys1108

Bjerhammar, A., 1985. On a Relativistic Geodesy. *Bulletin Géodésique*, 59(3): 207–220. https://doi.org/10.1007/bf02520327

Bloom, B. J., Nicholson, T. L., Williams, J. R., et al., 2014. An Optical Lattice Clock with Accuracy and Stability at the 10^{-18} Level. *Nature*, 506(7486): 71–75. https://doi.org/10.1038/nature12941

Chou, C. W., Hume, D. B., Koelemeij, J., et al., 2010a. Frequency Comparison of Two High-Accuracy Al^+ Optical Clocks. *Physical Review Letters*, 104(7): 070802. https://doi.org/10.1103/physrevlett.104.070802

Chou, C. W., Hume, D. B., Rosenband, T., et al., 2010b. Optical Clocks and Relativity. *Science*, 329(5999): 1630–1633. https://doi.org/10.1126/science.1192720

Diddams, S. A., Bergquist, J. C., Jefferts, S. R., et al., 2004. Standards of Time and Frequency at the Outset of the 21st Century. *Science*, 306(5700): 1318–1324. https://doi.org/10.1126/science.1102330

Diddams, S. A., Udem, T., Bergquist, J. C., et al., 2001. An Optical Clock Based on a Single Trapped $^{199}Hg^+$ Ion. *Science*, 293(5531): 825–828. https://doi.org/10.1126/science.1061171

Droste, S., Ozimek, F., Udem, T., et al., 2013. Optical-Frequency Transfer over a Single-Span 1 840 km Fiber Link. *Physical Review Letters*, 111(11): 110801. https://doi.org/10.1103/physrevlett.111.110801

Dziewonski, A. M., Anderson, D. L., 1981. Preliminary Reference Earth Model. *Physics of the Earth and Planetary Interiors*, 25(4): 297–356. https://doi.org/10.1016/0031-9201(81)90046-7

Flury, J., 2016. Relativistic Geodesy. *Journal of Physics Conference Series*, 723(1): 012051

Grosche, G., Terra, O., Predehl, K., et al., 2009. Optical Frequency Transfer via 146 km Fiber Link with 10^{-19} Relative Accuracy. *Optics Letters*, 34(15): 2270–2272. https://doi.org/10.13039/501100000844

Grotti, J., Koller, S., Vogt, S., et al., 2018. Geodesy and Metrology with a Transportable Optical Clock. *Nature Physics*, 14(5): 437–441. https://doi.org/10.1038/s41567-017-0042-3

Guena, J., Abgrall, M., Rovera, D., et al., 2012. Progress in Atomic Fountains at LNE-SYRTE. *Ultrasonics*, *Ferroelectrics and Frequency Control*, *IEEE Transactions on*, 59(3): 391–409. https://doi.org/10.1109/tuffc.2012.2208

Heiskanen, W. A., Moritz, H., 1967. Physical Geodesy. Freeman and Company, San Francisco

Hinkley, N., Sherman, J. A., Phillips, N. B., et al., 2013. An Atomic Clock with 10^{-18} Instability. *Science*, 341(6151): 1215–1218. https://doi.org/10.1126/science.1240420

Hofmann-Wellenhof, B., Moritz, H., 2006. Physical Geodesy. Springer

Huntemann, N., Okhapkin, M., Lipphardt, B., et al., 2012. High-Accuracy Optical Clock Based on the Octupole Transition in $^{171}Yb^{+}$. *Physical Review Letters*, 108(9): 090801. https://doi.org/10.1103/physrevlett.108.090801

Jiang, H., Kéfélian, F., Crane, S., et al., 2008. Long-Distance Frequency Transfer over an Urban Fiber Link Using Optical Phase Stabilization. *Journal of the Optical Society of America B*, 25(12): 2029–2035. https://doi.org/10.13039/501100001665

Katila, T., Riski, K. J., 1981. Measurement of the Interaction between Electromagnetic Radiation and Gravitational Field Using ^{67}Zn Mössbauer Spectroscopy. *Physics Letters A*, 83(2): 51–54. https://doi.org/10.1016/0375-9601(81)90062-1

Katori, H., 2011. Optical Lattice Clocks and Quantum Metrology. *Nature Photonics*, 5(4): 203–210. https://doi.org/10.1038/nphoton.2011.45

Kéfélian, F., Lopez, O., Jiang, H. F., et al., 2009. High-Resolution Optical Frequency Dissemination on a Telecommunications Network with Data Traffic. *Optics Letters*, 34(10): 1573–1575. https://doi.org/10.13039/501100001665

Li, W. Y., Liu, Y. X., Li, B., et al., 2016. Hydrocarbon Exploration in the South Yellow Sea Based on Airborne Gravity, China. *Journal of Earth Science*, 27(4): 686–698. https://doi.org/10.1007/s12583-015-0607-y

Lion, G. I., Panet, I., Wolf, P., et al., 2017. Determination of a High Spatial Resolution Geopotential Model Using Atomic Clock Comparisons. *Journal of Geodesy*, 91(6): 597–611. https://doi.org/10.13039/501100000781

Lisdat, C., Grosche, G., Quintin, N., et al., 2016. A Clock Network for Geodesy and Fundamental Science. *Nature Communications*, 7: 12443. https://doi.org/10.1038/ncomms12443

Lopez, O., Haboucha, A., Chanteau, B., et al., 2012. Ultra-Stable Long Distance Optical Frequency Distribution Using the Internet Fiber Network. *Optics Express*, 20(21): 23518. https://doi.org/10.1364/oe.20.023518

Lopez, O., Kanj, A., Pottie, P. E., et al., 2013. Simultaneous Remote Transfer of Accurate Timing and Optical Frequency over a Public Fiber Network. *Applied Physics B*, 110(1): 3–6. https://doi.org/10.1007/s00340-012-5241-0

Ludlow, A. D., Zelevinsky, T., Campbell, G. K., et al., 2008. Sr Lattice Clock at 1×10^{-16} Fractional Uncertainty by Remote Optical Evaluation with a Ca Clock. *Science*, 319(5871): 1805–1808. https://doi.org/10.1126/science.1153341

Ma, L. S., Bartels, A., Robertsson, L., et al., 2004. Optical Frequency Synthesis and Comparison with Uncertainty at the 10^{-19} Level. *Science*, 303(5665): 1843–1845. https://doi.org/10.1126/science.1095092

Ma, L. S., Jungner, P., Ye, J., et al., 1994. Delivering the Same Optical Frequency at Two Places: Accurate Cancellation of Phase Noise Introduced by an Optical Fiber or other Time-Varying Path. *Optics Letters*, 19(21): 1777–1779. https://doi.org/10.1364/ol.19.001777

Madej, A. A., Dubé, P., Zhou, Z. C., et al., 2012. $^{88}Sr^{+}$ 445-THz Single-Ion Reference at the 10^{-17} Level via Control and Cancellation of Systematic Uncertainties and Its Measurement against the SI Second. *Physical Review Letters*, 109(20): 203002. https://doi.org/10.1103/physrevlett.109.203002

Mai, E., 2013. Time, Atomic Clocks, and Relativistic Geodesy. Deutsche Geodätische Kommission, Reihe A, Theoretische Geodäsie, Heft Nr. 124, Verlag der Bayerischen Akademie der Wissenschaften, München

Marra, G., Slavík, R., Margolis, H. S., et al., 2011. High-Resolution Microwave Frequency Transfer over an 86-km-Long Optical Fiber Network Using a Mode-Locked Laser. *Optics Letters*, 36(4): 511. https://doi.org/10.13039/501100000821

Müller, H., Peters, A., Chu, S., 2010. A Precision Measurement of the Gravitational Redshift by the Interference of Matter Waves. *Nature*, 463(7283): 926–929. https://doi.org/10.1038/nature08776

Newbury, N. R., Swann, W. C., Coddington, I., et al., 2007a. Fiber Laser-Based Frequency Combs with High Relative Frequency Stability. Frequency Control Symposium, 2007 Joint with the 21st European Frequency and Time Forum. IEEE International. 980–983. https://doi.org/10.1109/FREQ.2007.4319226

Newbury, N. R., Williams, P. A., Swann, W. C., 2007b. Coherent Transfer of an Optical Carrier over 251 km. *Optics Letters*, 32(21): 3056–3058. https://doi.org/10.1364/ol.32.003056

Pound, R. V., Rebka, G. A. Jr., 1959. Gravitational Red-Shift in Nuclear Resonance. *Physical Review Letters*, 3(9): 439–441. https://doi.org/10.1103/physrevlett.3.439

Pound, R. V., Rebka, G. A. Jr., 1960a. Attempts to Detect Resonance Scattering InZn67; The Effect of Zero-Point Vibrations. *Physical Review Letters*, 4(8): 397–399. https://doi.org/10.1103/physrevlett.4.397

Pound, R. V., Rebka, G. A. Jr., 1960b. Variation with Temperature of the Energy of Recoil-Free Gamma Rays from Solids. *Physical Review Letters*, 4(6): 274–275. https://doi.org/10.1103/physrevlett.4.274

Pound, R. V., Snider, J. L., 1965. Effect of Gravity on Gamma Radiation. *Physical Review*, 140(3B): B788–B803. https://doi.org/10.1103/physrev.140.b788

Predehl, K., Grosche, G., Raupach, S. M. F., et al., 2012. A 920-Kilometer Optical Fiber Link for Frequency Metrology at the 19th Decimal Place. *Science*, 336(6080): 441–444. https://doi.org/10.1126/science.1218442

Primas, L. E., Lutes, G. F., Sydnor, R. L., 1988. Fiber Optic Frequency Transfer Link. Proceedings of 42nd Annual Symposium on Frequency Control, June 1–3, 1988, Baltimore, MD. 478–484

Raupach, S. M. F., Grosche, G., 2013. Chirped Frequency Transfer with an Accuracy of 10^{-18} and Its Application to the Remote Synchronization of Timescales. arXiv: 1308.6725v2 [physics.optics] (2013-9-30)

Raupach, S. M. F., Koczwara, A., Grosche, G., 2014. Optical Frequency Transfer via a 660 km Underground Fiber Link Using a Remote Brillouin Amplifier. *Optics Express*, 22(22): 26537–26547. https://doi.org/10.1364/oe.22.026537

Rosenband, T., Hume, D. B., Schmidt, P. O., et al., 2008. Frequency Ratio of Al^{+} and Hg^{+} Single-Ion Optical Clocks, Metrology at the 17th Decimal Place. *Science*, 319(5871): 1808–1812. https://doi.org/10.1126/science.1154622

Shen, W.-B., 1998. Relativistic Physical Geodesy: [Dissertation]. Graz Technical University, Graz

Shen, W.-B., 2013a. Orthometric Height Determination Based upon Optical Clocks and Fiber Frequency Transfer Technique. 2013 Saudi International Electronics, Communications and Photonics Conference (SIECPC), April 27–30, 2013, Riyadh, Saudi Arabia. https://doi.org/10.1109/SIECPC.2013.6550987

Shen, W.-B., 2013b. Orthometric Height Determination Using Optical Clocks. *EGU General Assembly Conference Abstracts*, 15: 5214

Shen, W.-B., Chao, D., Jin, B., 1993. On Relativistic Geoid. *Bollettino di Geodesia e Scienze Affini*, *52*(3): 207–216

Shen, W.-B., Ning, J. S., Chao, D. B., et al., 2009. A Proposal on the Test of General Relativity by Clock Transportation Experiments. *Advances in Space Research*, 43(1): 164–166. https://doi.org/10.1016/j.asr.2008.04.001

Shen, W.-B., Ning, J. S., Liu, J. N., et al., 2011. Determination of the Geopotential and Orthometric Height Based on Frequency Shift Equation. *Natural Science*, 3(5): 388–396. https://doi.org/10.4236/ns.2011.35052

Shen, W.-B., Peng, Z., 2012. Gravity Potential Determination Using Remote Optical Fiber. International Symposium on Gravity, Geoid and Height Systems GGHS 2012. Dec. 3, 2012, Venice, Italy

Shen, Z. Y., Shen, W.-B., Zhang, S. X., 2016. Formulation of Geopotential Difference Determination Using Optical-Atomic Clocks Onboard Satellites and on Ground Based on Doppler Cancellation System. *Geophysical Journal International*, 206(2): 1162–1168. https://doi.org/10.1093/gji/ggw198

Shen, Z. Y., Shen, W.-B., Zhang, S. X., 2017. Determination of Gravitational Potential at Ground Using Optical-Atomic Clocks on Board Satellites and on Ground Stations and Relevant Simulation Experiments. *Surveys in Geophysics*, 38(4): 757–780. https://doi.org/10.1007/s10712-017-9414-6

Snider, J. L., 1972. New Measurement of the Solar Gravitational Red Shift. *Physical Review Letters*, 28(13): 853–856. https://doi.org/10.1103/physrevlett.28.853

Soffel, M., Herold, H., Ruder, H., et al., 1988a. Relativistic Geodesy: The Concept of Asymptotically Fixed Reference Frames. *Manuscr. Geod.*, 13(3): 139–142

Soffel, M., Herold, H., Ruder, H., et al., 1988b. Relativistic Theory of Gravimetric Measurements and Definition of the Geoid. *Manuscr. Geod.*, 13: 143–146

Takano, T., Takamoto, M., Ushijima, I., et al., 2016. Geopotential Measurements with Synchronously Linked Optical Lattice Clocks. *Nature Photonics*, 10(10): 662–666. https://doi.org/10.1038/nphoton.2016.159

Tenzer, R., Bagherbandi, M., 2016. Theoretical Deficiencies of Isostatic Schemes in Modeling the Crustal Thickness along the Convergent Continental Tectonic Plate Boundaries. *Journal of Earth Science*, 27(6): 1045–1053. https://doi.org/10.1007/s12583-015-0608-x

Turneaure, J. P., Will, C. M., Farrell, B. F., et al., 1983. Test of the Principle of Equivalence by a Null Gravitational Red-Shift Experiment. *Physical Review D*, 27(8): 1705–1714. https://doi.org/10.1103/physrevd.27.1705

Ushijima, I., Takamoto, M., Das, M., et al., 2015. Cryogenic Optical Lattice Clocks. *Nature Photonics*, 9(3): 185–189. https://doi.org/10.1038/nphoton.2015.5

Vessot, R. F. C., Levine, M. W., 1979. A Test of the Equivalence Principle Using a Space-Borne Clock. *General Relativity and Gravitation*, 10(3): 181–204. https://doi.org/10.1007/bf00759854

Vessot, R. F. C., Levine, M. W., Mattison, E. M., et al., 1980. Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser. *Physical Review Letters*, 45(26): 2081–2084. https://doi.org/10.1103/physrevlett.45.2081

Wada, M., Watabe, K.-I., Okubo, S., et al., 2015. A Precise Frequency Comparison System Using an Optical Carrier. *Electronics and Communications in Japan*, 98: 19–27

Weinberg, S., 1972. Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. Wiley, New York

Ye, J., Peng, J.-L., Jones, R. J., et al., 2003. Delivery of High-Stability Optical and Microwave Frequency Standards over an Optical Fiber Network. *Journal of the Optical Society of America B*, 20(7): 1459. https://doi.org/10.1364/josab.20.001459